

UČNI LIST – Integral

1) Integriraj:

a) $\int \frac{\sqrt[4]{x} dx}{x} =$

b) $\int \frac{x dx}{\sqrt[3]{x}} =$

c) $\int \frac{\sqrt[3]{x} dx}{\sqrt[6]{x^5}} =$

d) $\int \frac{\sqrt[3]{x^2} dx}{\sqrt[4]{x^3} \cdot \sqrt[12]{x^{11}}} =$

e) $\int \frac{x^2 - 1}{\sqrt{x}} dx =$

f) $\int \frac{\sqrt[3]{x^4} - \sqrt[3]{x}}{\sqrt[3]{x^4}} dx =$

2) Integriraj:

a) $\int \frac{x^4 + 2x^3 - 5x}{x^2} dx =$

b) $\int \frac{x^4 - x^3 + 2x}{x^4} dx =$

c) $\int \frac{2x^4 - x^3 - x}{x^3} dx =$

d) $\int \frac{x^5 - 4x^2 + 2}{x^3} dx =$

3) Integriraj:

a) $\int \frac{28x^5 - 12x^4 - 5x}{x^2} dx =$

b) $\int \frac{12 \cdot x^5 - 9 \cdot \sqrt[4]{x^7}}{x^2} dx =$

c) $\int \frac{20 \cdot x^4 - 8 \cdot \sqrt[3]{x^4}}{x} dx =$

d) $\int \frac{6 \cdot x^3 - 9 \cdot \sqrt[5]{x^3}}{x} dx =$

4) Integriraj:

a) $\int \frac{8 \cdot x^4 - 6 \cdot \sqrt[3]{x^2}}{x} dx =$

b) $\int \frac{12x^5 - \sqrt[6]{x^5} + 3x}{x^2} dx =$

c) $\int \frac{15x^4 - \sqrt[4]{x^3} + 2x}{x^2} dx =$

d) $\int \frac{9 \cdot x^4 - 6 \cdot \sqrt[5]{x^7} - 2 \cdot x}{x^2} dx =$

5) Integriraj s substitucijo:

a) $\int (4x - 2)^5 dx =$

b) $\int \sqrt{3x + 2} dx =$

c) $\int \sqrt[4]{\frac{x-3}{2}} dx =$

d) $\int \frac{x dx}{2x^2 - 3} =$

6) Integriraj s substitucijo:

a) $\int \sqrt[3]{x^2 - 5} \cdot x dx =$

b) $\int \frac{x dx}{3x^2 + 4} =$

c) $\int \frac{12x^2 dx}{(x^3 - 3)^3} =$

d) $\int \frac{8x dx}{\sqrt[3]{x^2 - 4}} =$

7) Integriraj s substitucijo:

a) $\int \frac{2x+3}{(x^2+3x-4)^2} dx =$

b) $\int \frac{x+3}{x^2+6x+8} dx =$

c) $\int \frac{x+2}{x^2+4x+3} dx =$

d) $\int \frac{x^3-x}{x^4-2x^2} dx =$

8) Integriraj s substitucijo:

a) $\int \frac{\cos 2x}{\cos^2 x} dx =$

b) $\int \frac{\sin 2x}{\sin^4 x} dx =$

c) $\int \frac{\sin 2x}{\cos^4 x} dx =$

d) $\int \frac{1-\sin x}{\cos^2 x} dx =$

9) Integriraj *Per partes*:

a) $\int (4x-5) \cdot e^x dx =$

b) $\int x \cdot \ln x dx =$

c) $\int (3x+1) \cdot e^x dx =$

d) $\int x^4 \cdot \ln x dx =$

e) $\int (2x+3) \cdot \cos x dx =$

f) $\int (5x-2) \cdot \sin x dx =$

10) Izračunaj določeni integral:

a) $\int_0^2 x^3 dx =$

b) $\int_{-2}^1 x^5 dx =$

c) $\int_2^{10} \frac{dx}{x} =$

d) $\int_1^4 (x^2-6x) dx =$

e) $\int_{-2}^3 (8x^3-4x) dx =$

f) $\int_1^2 \frac{10x^2-2}{x} dx =$

11) Izračunaj določeni integral s substitucijo:

a) $\int_1^2 (3x-1)^3 dx =$

b) $\int_0^1 \frac{dx}{(4x+2)^2} =$

c) $\int_1^5 \frac{dx}{6x-3} =$

d) $\int_1^3 \sqrt{2x+2} dx =$

e) $\int_0^5 \frac{dx}{\sqrt{9-x}} =$

f) $\int_0^{\frac{\pi}{2}} \sin^4 x \cdot \cos x \cdot dx =$

- 12) Poišči teme in nariši graf funkcije $f(x) = -x^2 + x + 2$. Izračunaj ploščino lika, ki ga omejujeta graf funkcije in abscisna os.
- 13) Poišči teme in nariši graf funkcije $f(x) = x - x^2$. Izračunaj ploščino lika, ki ga omejujeta graf funkcije in abscisna os.
- 14) Skiciraj lik, ki ga omejujejo graf funkcije $f(x) = x^3 - 3x^2 + x + 5$, abscisna os in premica $x = 3$, nato pa izračunaj njegovo ploščino.
- 15) Skiciraj lik, ki ga omejujejo graf funkcije $f(x) = x^3 - 8x^2 + 21x - 14$, abscisna os in premica $x = 4$, nato pa izračunaj njegovo ploščino.
- 16) Poišči ničle polinoma in izračunaj ploščino lika, ki ga omejujeta polinom in abscisna os:
- a) $p(x) = x^3 + 2x^2$
 - b) $p(x) = x^3 - 8x^2 + 16x$
 - c) $p(x) = x^3 - 4x^2$
- 17) Izračunaj ploščino lika, ki ga omejujeta grafa funkcij:
- a) $f(x) = -x^2 + 2x$, $g(x) = x - 2$
 - b) $f(x) = -x^2 + 2x + 3$, $g(x) = 2x - 1$
 - c) $f(x) = 2x^2 - 8x + 6$, $g(x) = -x + 3$
 - d) $f(x) = -x^2 + 9$, $g(x) = x^2 + 2x - 3$

REŠITVE UČNEGA LISTA – Integral

1) a) $4 \cdot \sqrt[4]{x} + C$

b) $\frac{3 \cdot \sqrt[3]{x^5}}{5} + C$

c) $2 \cdot \sqrt{x} + C$

d) $\ln|x| + C$

e) $\frac{2 \cdot \sqrt{x^5}}{5} - 2 \cdot \sqrt{x} + C$

f) $x - \ln|x| + C$

2) a) $\frac{x^3}{3} + x^2 - 5 \cdot \ln|x| + C$

b) $x - \ln|x| - \frac{1}{x^2} + C$

c) $x^2 - x + \frac{1}{x} + C$

d) $\frac{x^3}{3} - 4 \cdot \ln|x| - \frac{1}{x^2} + C$

3) a) $7 \cdot x^4 - 4 \cdot x^3 - 5 \cdot \ln|x| + C$

b) $3 \cdot x^4 - 12 \cdot \sqrt[4]{x^3} + C$

c) $5 \cdot x^4 - 6 \cdot \sqrt[3]{x^4} + C$

d) $2 \cdot x^3 - 15 \cdot \sqrt[5]{x^3} + C$

4) a) $2 \cdot x^4 - 9 \cdot \sqrt[3]{x^2} + C$

b) $3 \cdot x^4 + \frac{6}{\sqrt[6]{x}} + \ln|x|^3 + C$

c) $5 \cdot x^3 + \frac{4}{\sqrt[4]{x}} + \ln|x|^2 + C$

d) $3 \cdot x^3 - 15 \cdot \sqrt[5]{x^2} - \ln|x|^2 + C$

5) a) $\frac{(4x-2)^6}{24} + C$

b) $\frac{2 \cdot \sqrt{(3x+2)^3}}{9} + C$

c) $\frac{8}{5} \cdot \sqrt[4]{\left(\frac{x-3}{2}\right)^5} + C$

d) $\frac{\ln|2x^2-3|}{4} + C$

6) a) $\frac{3 \cdot \sqrt[3]{(x^2-5)^4}}{8} + C$

b) $\frac{\ln|3x^2+4|}{6} + C$

c) $-\frac{2}{(x^3-3)^2} + C$

d) $6 \cdot \sqrt[3]{(x^2-4)^2} + C$

7) a) $-\frac{1}{x^2+3x-4} + C$

b) $\frac{\ln|x^2+6x+8|}{2} + C$

c) $\frac{\ln|x^2+4x+3|}{2} + C$

d) $\frac{\ln|x^4-2x^2|}{4} + C$

8) a) $2x - \tan x + C$

b) $-\frac{1}{\sin^2 x} + C$

c) $\frac{1}{\cos^2 x} + C$

d) $\tan x - \frac{1}{\cos x} + C$

9) a) $(4x-9) \cdot e^x + C$

b) $\frac{x^2 \cdot \ln x}{2} - \frac{x^2}{4} + C$

c) $(3x-2) \cdot e^x + C$

d) $\frac{x^5 \cdot \ln x}{5} - \frac{x^5}{25} + C$

e) $(2x+3) \cdot \sin x + 2 \cdot \cos x + C$

f) $(2-5x) \cdot \cos x + 5 \cdot \sin x + C$

10) a) 4

b) $-\frac{63}{6}$

c) $\ln 5$

d) -24

e) 116

f) $15 - \ln 4$

11) a) $50\frac{3}{4}$

b) $\frac{1}{12}$

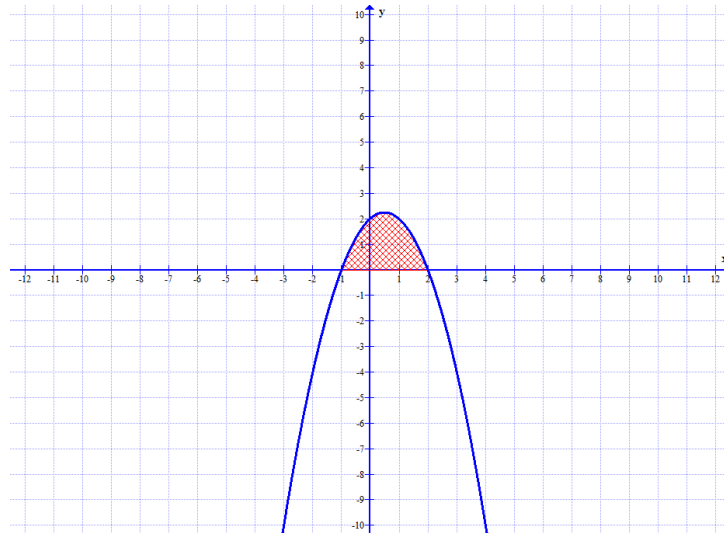
c) $\ln \sqrt[3]{3}$

d) $\frac{16\sqrt{2}-8}{3}$

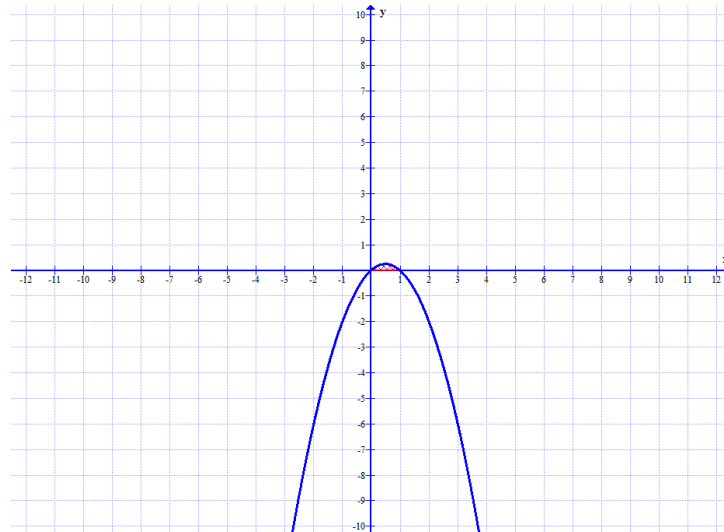
e) 2

f) $\frac{1}{5}$

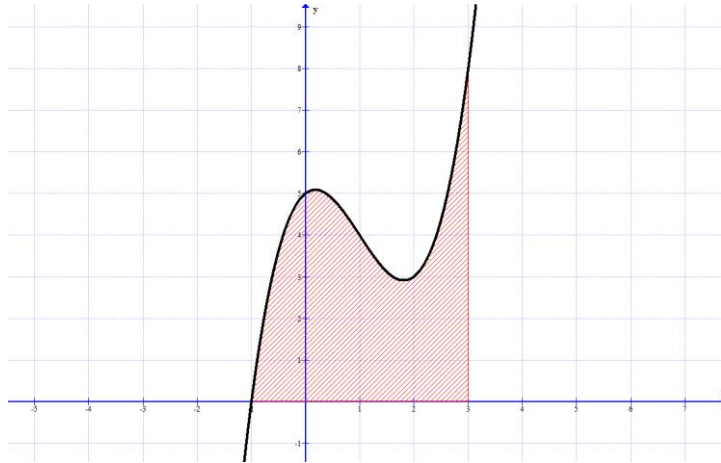
12) $T\left(\frac{1}{2}, \frac{9}{4}\right), S = \int_{-1}^2 (-x^2 + x + 2) dx = \frac{9}{2}$



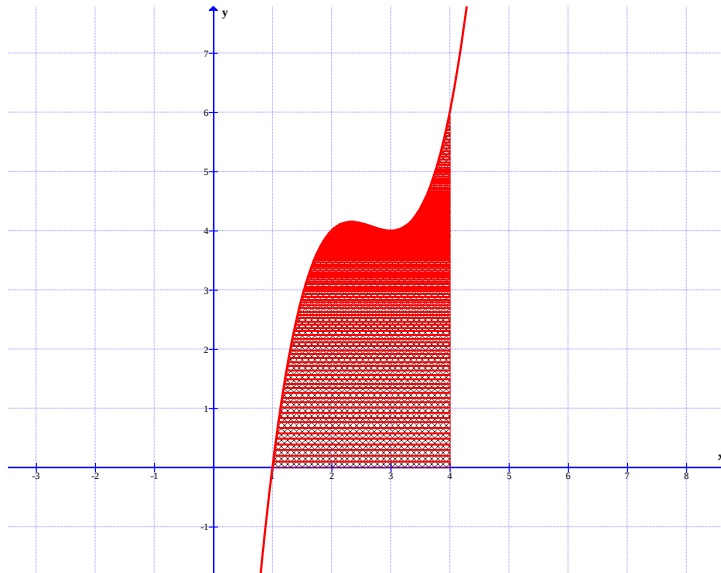
13) $T\left(\frac{1}{2}, \frac{1}{4}\right), S = \int_0^1 (x - x^2) dx = \frac{1}{6}$



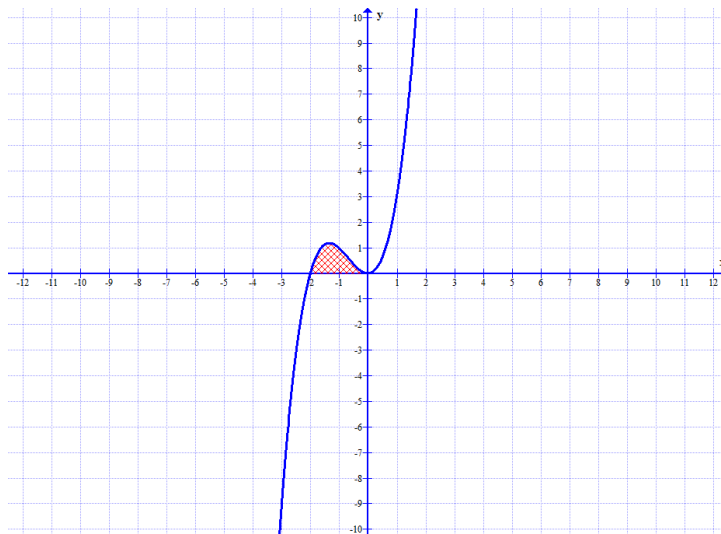
$$14) \quad S = \int_{-1}^3 (x^3 - 3x^2 + x + 5) dx = 16$$



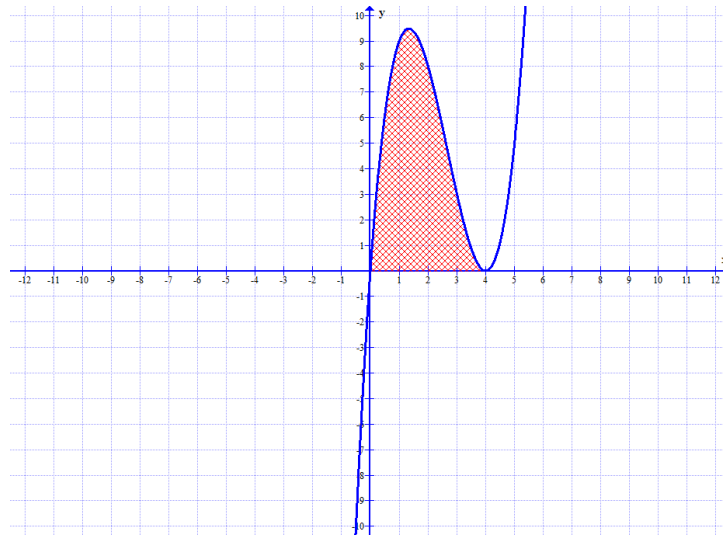
$$15) \quad S = \int_1^4 (x^3 - 8x^2 + 21x - 14) dx = 11\frac{1}{4}$$



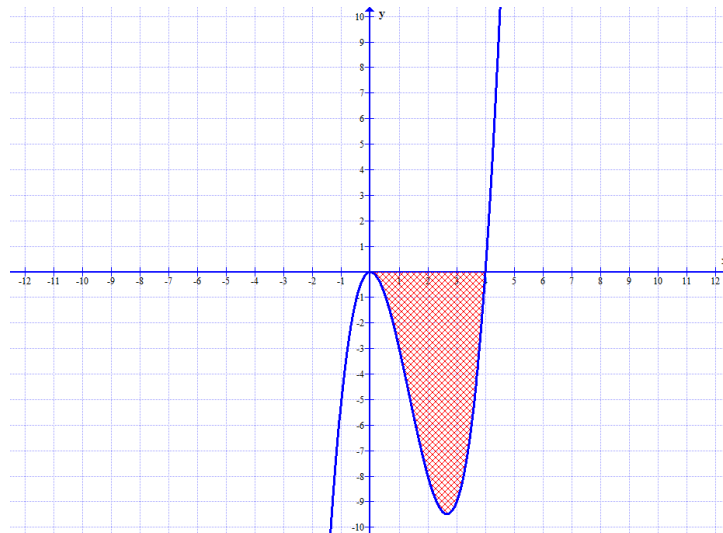
$$16) \quad a) \quad S = \int_{-2}^0 (x^3 + 2x^2) dx = \frac{4}{3}$$



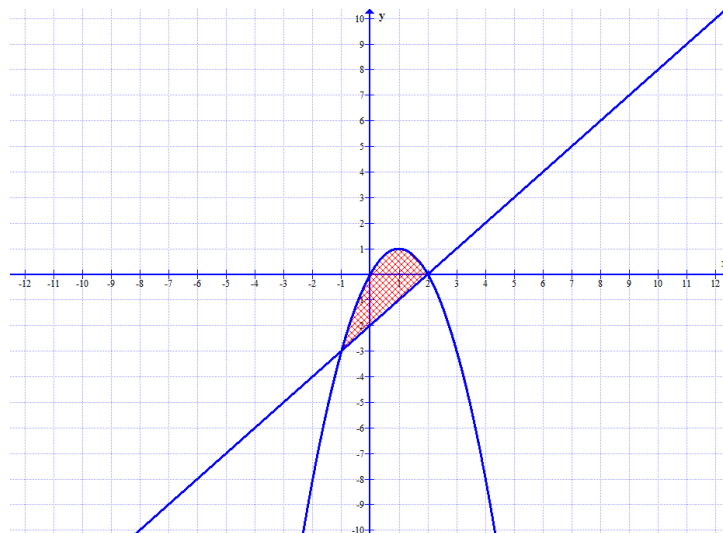
b) $S = \int_0^4 (x^3 - 8x^2 + 16x) dx = \frac{64}{3}$



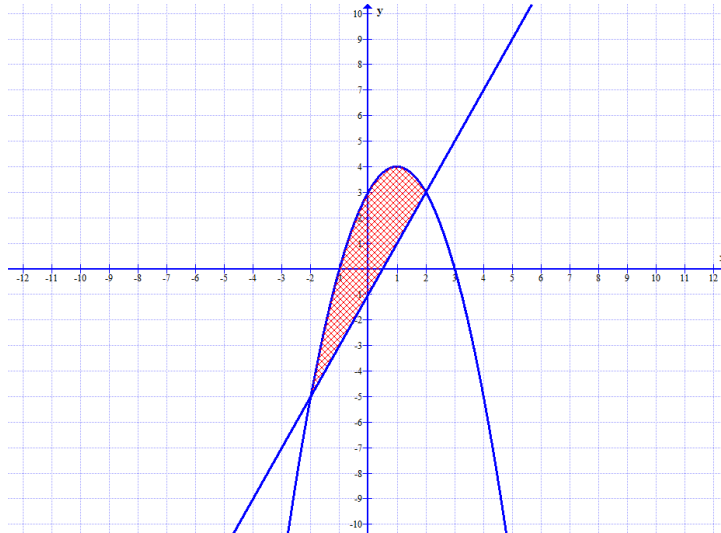
c) $S = \int_0^4 (x^3 - 4x^2) dx = \frac{64}{3}$



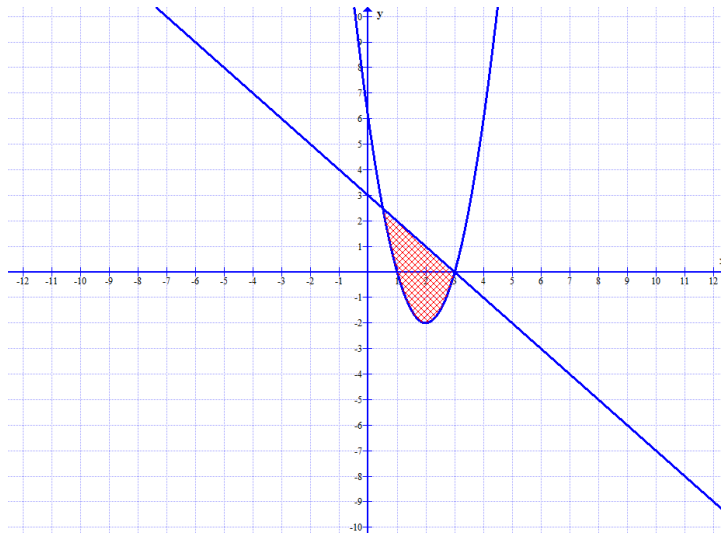
17) a) $S = \int_{-1}^2 (-x^2 + x + 2) dx = \frac{9}{2}$



$$\text{b) } S = \int_{-2}^2 (-x^2 + 4) dx = \frac{32}{3}$$



$$\text{c) } S = \int_{\frac{1}{2}}^3 (-2x^2 + 7x - 3) dx = \frac{125}{24}$$



$$\text{d) } S = \int_{-3}^2 (-2x^2 - 2x + 12) dx = \frac{122}{3}$$

